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Holographic Complexity for Accelerating AdS Black holes

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Introduction

In 1970s, Hawking and Beckenstein discovered the thermodynamic properties of black holes. They attributed entropy to black holes that is defined as follows [1,2]

$$S_{BH} = \frac{Area}{4G}, \qquad (1)$$

where Area is the surface of the black hole. This relationship has a subtle physical interpretation. From thermodynamics, we know that the entropy of the system is proportional to its volume, so there is a relationship between the volume of the system and the area in Eq. (1). We have to make a connection between these two geometric parts by entropy. Therefore, it is interpreted as follows: The number of degrees of freedom of a gravitational system in D-dimension is equal to the number of degrees of freedom of a non-gravitational system (theory) in (D-1)-dimension.

Maldacena in 1999, using string theory, was able to find an example of this duality theoretically [3]. He showed that the non-gravitational physical system must be a gauge theory. Now the interpretation is as follows: the partition function of a theory of gravity in D-dimension equivalent to the partition function of a gauge theory in (D −1)-dimension

$$Z^{d}_{\text{gravity}} = Z^{d-1}_{gauge}, \qquad (2)$$

where Z is the system partition function and indicates the number of degrees of system freedom [4]. This theory can be explained in geometric language as follows:

At the zero temperature state

[ground state in boundary] ~ [pure AdS field in bulk] (3)

And at a limited temperature

[An excited state in volume] ~ [AdS field and a field in bulk] (4)

In general, we can say that "every geometry is equivalent to a state". Information and com- putation theory as a branch of mathematical and computer science have rapid and increas- ing impact on research in various branches of physics, especially gravity-quantum. Entropy and complexity are two key concepts in information theory that have recently played a key role in understanding the physics of black holes in the context of the AdS/CFT duality framework. In the following, we intend to deal with holographic complexity.

Holographic complexity

Complexity is a valuable concept in various branches of physics, including black hole physics. The AdS/CFT duality suggests a correspondence between a geometric quantity in the bulk and a boundary quantity. When complexity is viewed as a boundary quantity, it prompts the question of its dual in the bulk. One strong candidate for this dual is the classical action, leading to the "complexity-action" duality defined as $\boldsymbol{C} = \frac{\boldsymbol{S_W}}{\boldsymbol{\pi \hbar}}$ [7,8]. Here, the classical action for the Wheeler-DeWitt patch correlates with the complexity of the boundary state.

The Wheeler-DeWitt patch encompasses all space-like paths connecting selected time pairs from the left and right boundaries. This duality implies that the action for this patch is equivalent to the complexity at either boundary. However, challenges arise in determining the nature of the bound, as Susskind and colleagues debated whether it should be space-like or time-like. This led them to incorporate the York-Gibbons-Hawking surface action into their calculations and to focus on the "rate of growth of complexity" rather than direct complexity computation. Others, through careful boundary analysis, calculated complexity and found their results aligned with those of Susskind et al [9].

The Rate of Growth of Complexity for Accelerating Black holes

The rate of growth of complexity has been studied for different types of AdS black holes [10, 11], and we intend to do this for accelerated black holes.

Typically, the metric of slowly accelerating black holes with a cosmological constant is represented by the following metric [12]:

$$ds^2 = \frac{1}{\Omega^2}\left[f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(\frac{d\theta^2}{g(\theta)} + g(\theta)\sin\theta^2\frac{d\phi^2}{K^2}\right)\right] \qquad (5)$$

Where

$$\Omega = 1 + A\,r\,\cos\theta,$$

$$\mathrm{f(r)} = (1 - \mathrm{A^2 r^2})\left(1 - \frac{2\,\mathrm{m}}{\mathrm{r}}\right) + \frac{\mathrm{r^2}}{\ell^2} \qquad (6)$$

$$g(\theta) = 1 + 2\,m\,A\,\cos\theta,$$

In this section we are going to obtain the complexity growth according to "complex-ity=action" conjecture (CA) in the accelerated black holes. The general action for the Wheeler-DeWitt patch is written as follows

$$S_W = S_{EH} + S_{YGH}$$

The above three terms are the Einstein-Hilbert (EH) action and the York-Gibbons-Hawking (YGH) surface term respectively.

The EH action in D-dimensional space-time is as follows

$$S_{EH} = \frac{1}{16\,\pi\,G}\int_V \sqrt{-|g|}(R - 2\,\Lambda)d^D x. \qquad (7)$$

where includes a (negative) cosmological constant, Λ. In general, the relation between Ricci scalar and cosmology constant is as follow:

$$R - 2\,\Lambda = \frac{-6}{\ell^2}. \qquad (8)$$

Then the action of The Einstein-Hilbert action (EH) is as follows:

$$\frac{dS_{EH}}{dt} = \frac{-6}{16\,\pi\,G\,\ell^2}\int_0^{r_h}\int_0^{2\,\pi}\int_{\pi/2}^{\pi}\frac{r^2\sin\theta}{K(1+Ar\cos\theta)^4}d\theta d\phi dr, \qquad (9)$$

A York-Gibbons-Hawking (YGH) surface term is as below:

$$S_{YGH} = \frac{1}{8\,\pi\,G}\int_\Sigma \sqrt{-|h|}\kappa d^{D-1}x, \qquad (10)$$

where κ is the extrinsic curvature tensor .

$$\frac{dS_{YGH}}{dt} = \frac{1}{8\,\pi\,G}\int_0^{2\,\pi}\int_{\pi/2}^{\pi}\left(\frac{2rf(r)\sin\theta}{K\Omega^2} + \frac{r^2 f'\sin\theta}{2K\Omega^2} - \frac{3\Omega' f(r) r^2\sin\theta}{K\Omega^3}\right)d\theta d\phi. \qquad (11)$$

By summing the two relations (15) and (18), numerical calculations show that the rate of growth of complexity is inversely related to the acceleration, and in general it can be said that: the higher the degree of freedom of the black hole, the lower the rate of growth of complexity.

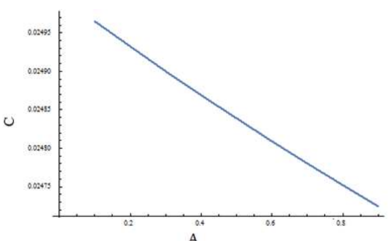

Figure 2. Growth rate behavior of complexity $\dot{C}$ in terms of black hole acceleration.

Conclusions

The results of this paper indicate the existence of an inverse relationship between the growth rate of complexity and the degrees of freedom of accelerated black holes. This findings allow us to gain a deeper understanding of the dynamic behavior of gravitational systems and their information structure. Also, the results can help develop new theories in the field of Theoretical Physics and cosmology and provide new solutions for future research in the field of holographic complexity and its connections to information theory. Finally, the importance of symmetry in the complexity behavior of black holes is clearly evident, and this can be the basis for further research in this area

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