A Higher-Dimensional Approach to the Hubble Constant Anomaly

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INTRODUCTION

Hubble Tension: The Hubble tension is the discrepancy between the early-universe (Planck) and late-universe (SH0ES) measurements of the Hubble constant. This unresolved conflict suggests potential new physics beyond the standard cosmological model.

Bulk-Brane Interaction: In the brane cosmology model, our observable universe is a 3-dimensional membrane (brane) embedded within a higher -dimensional space called the bulk. While all known particles and forces are confined to the brane, gravity can propagate throughout the entire bulk.

Power-Law Potential: A power-law potential, such as $V(\phi) \propto \frac{\phi^{2n}}{2^n}$,is a model for quintessence dark energy where a scalar field slowly rolls down the potential. This rolling generates a negative pressure that drives the observed accelerated expansion of the universe.

BULK-BRANE INTERACTION

The action:

$$S = \frac{1}{2}\int d^5 s\sqrt{-g}(R - g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi - 2V(\phi)) + \int d^4 s L_m\,(\psi_m, A^2(\phi)h_{\mu\nu})$$

The space-time metrics:

$$dS^2 = g_{\alpha\beta}dx^\alpha dx^\beta \;= h_{\mu\nu}dx^\mu dx^\nu + b^2(t,y)dy^2$$

Relation of variation in the brane tensor and bulk with respect to ϕ, by applying Bianchi identities

$$\nabla_\alpha T^{\alpha\beta}|brane = -\nabla_\alpha T^{\alpha\beta}|bulk = \left(\frac{d\ln A\,(\phi)}{d\phi}\right)T|brane\nabla^\beta\phi$$

The five-dimensional Friedmann-Robertson-Walker metric:

$$dS^2 = -n^2(t,y)dt^2 + a^2(t,y)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] + b^2(t,y)dy^2$$

The Friedman equation:

$$\dot{H} + H^2 = \frac{1}{12}\,[\frac{\zeta(1-3\omega_b)\rho_b}{H} - (\rho_\phi + 3P_\phi)] - \frac{1}{36}{\rho_b}^2\,[\frac{\zeta(1-3\omega_b)}{H} + 3\omega_b + 2]$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 4\zeta\rho_b$$

$$\dot{\rho_\phi} + 3H(1+\omega_\phi)\rho_\phi = 4\zeta\dot{\phi}\rho_b$$

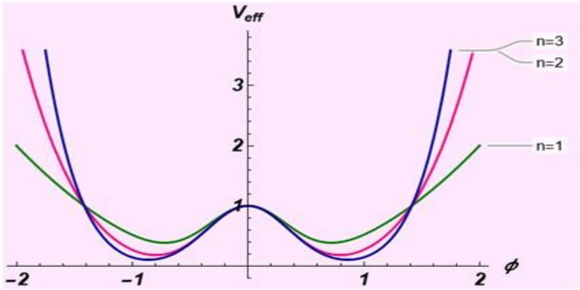
$$\dot{\rho_b} = -4\zeta\dot{\phi}\rho_b \;\rightarrow\; \rho_b = \rho_{b_0}e^{-4\zeta\phi} \;\xrightarrow{\rho_b = \rho_m + \rho_\Lambda \;\rightarrow\; \rho_m \ll \rho_\Lambda \;\rightarrow\; \rho_b \approx \rho_\Lambda}\; \rho_\Lambda = \rho_{\Lambda_0}e^{-4\zeta\phi}$$

The Dark Energy Model

Power-Low Potential:

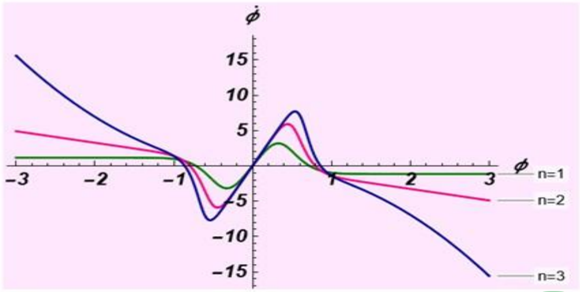
$$V_{eff} = V_0\frac{\phi^{2n}}{2^n} + \rho_{\Lambda_0}e^{-4\zeta\phi}$$

$$\zeta \equiv \phi$$



Inflation on the Brane:

$$\dot{\phi} = \frac{-n2^{1-n}\phi^{2n-1}V_0 + 8e^{-4\phi^2}\phi\rho_{\Lambda_0}}{3\sqrt{\frac{V_0}{6}2^{-n}\phi^{2n} - \frac{{\rho_{\Lambda_0}}^2}{36}e^{-8\phi^2}}}$$



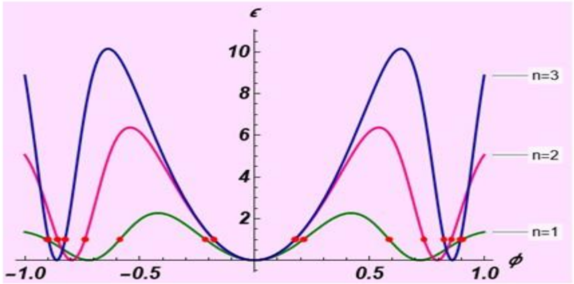
Inflationary Parameters

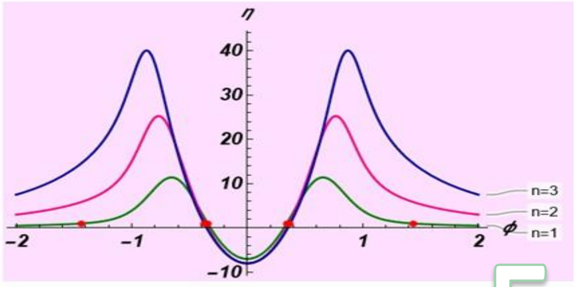
The first slow-roll inflation parameter:

$$\varepsilon = \frac{1}{2}\left(\frac{n2^{1-n}\phi^{2n-1}V_0 - 8e^{-4\phi^2}\phi\rho_{\Lambda_0}}{2^{-n}\phi^{2n}V_0 - e^{-4\phi^2}\rho_{\Lambda_0}}\right)^2$$

The second slow-roll inflation parameter:

$$\eta = \frac{(2n-1)n2^{1-n}\,\phi^{2n-2}V_0 - 8e^{-4\phi^2}(8\phi^2-1)\rho_{\Lambda_0}}{2^{-n}\phi^{2n}V_0 - e^{-4\phi^2}\rho_{\Lambda_0}}$$





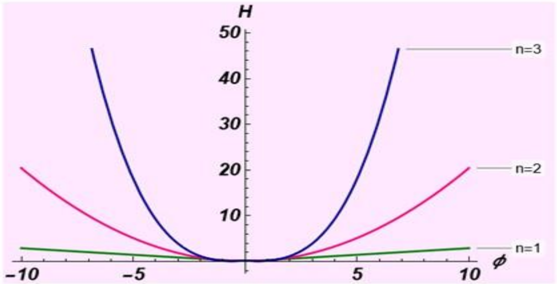
Hubble Parameter Analysis

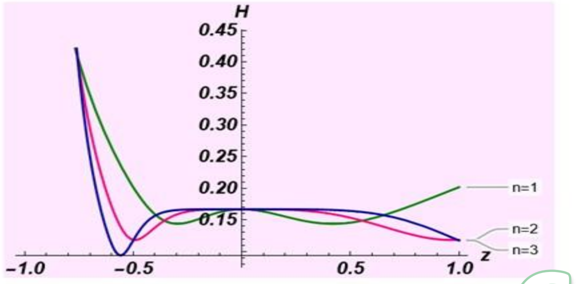
Hubble parameter derived from the Friedmann equations:

$$H^2 = \frac{V_0}{6}\frac{\phi^{2n}}{2^n} + \frac{\rho_{\Lambda_0}}{36}e^{-8\phi^2}$$

Hubble parameter based on the scale factor:

$$\phi = \alpha \ln a\,, \qquad a = \frac{1}{1+z}$$





Inflationary Parameters

The Tensor-to-Scalar :

$$r = 8\left(\frac{n2^{1-n}\phi^{2n-1}V_0 - 8e^{-4\phi^2}\phi\rho_{\Lambda_0}}{2^{-n}\phi^{2n}V_0 - e^{-4\phi^2}\rho_{\Lambda_0}}\right)^2$$

The spectral index:

$$n_s = 1 + \frac{(2n-1)n2^{2-n}\,\phi^{2n-2}V_0 - 16e^{-4\phi^2}(8\phi^2-1)\rho_{\Lambda_0}}{2^{-n}\phi^{2n}V_0 - e^{-4\phi^2}\rho_{\Lambda_0}} - 3\left(\frac{n2^{1-n}\phi^{2n-1}V_0 - 8e^{-4\phi^2}\phi\rho_{\Lambda_0}}{2^{-n}\phi^{2n}V_0 - e^{-4\phi^2}\rho_{\Lambda_0}}\right)^2$$

	$\varepsilon < 1$	$\lvert\eta\rvert < 1$	ε	η	r	$n_s \approx 1$
n=1	$-0.2150 < \phi < 0.2138$	$0.2938 < \phi < 0.3422$	8.8×10^{-6}	-0.02743	1.4×10^{-3}	0.94507
n=2	$-0.1775 < \phi < 0.1778$	$0.3194 < \phi < 0.3613$	4.1×10^{-6}	-0.03384	6.6×10^{-5}	0.93229
n=3	$-0.1775 < \phi < 0.1778$	$0.3287 < \phi < 0.3724$	7.2×10^{-7}	-0.04274	1.1×10^{-5}	0.91450

Results

- Inflation is driven by a self-interacting bulk scalar field non-minimally coupled to the brane.
- Despite modified brane cosmology, the slow-roll conditions remain four-dimensional, while energy transfers occur due to the coupling.
- The power-Low potential matches well with the Hubble Tension in Late times.

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