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Simulating Bubbles in a Holographic QCD-like Phase Diagram

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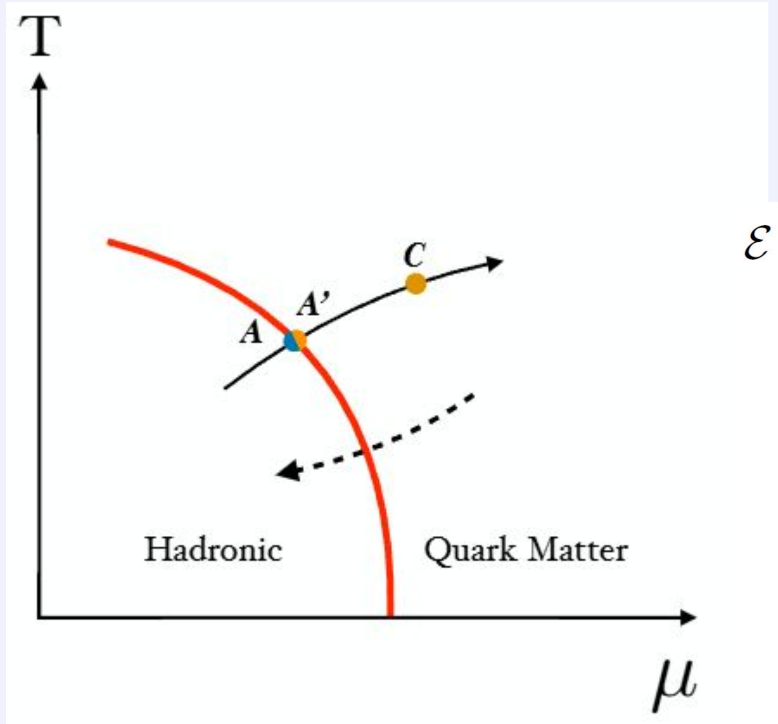
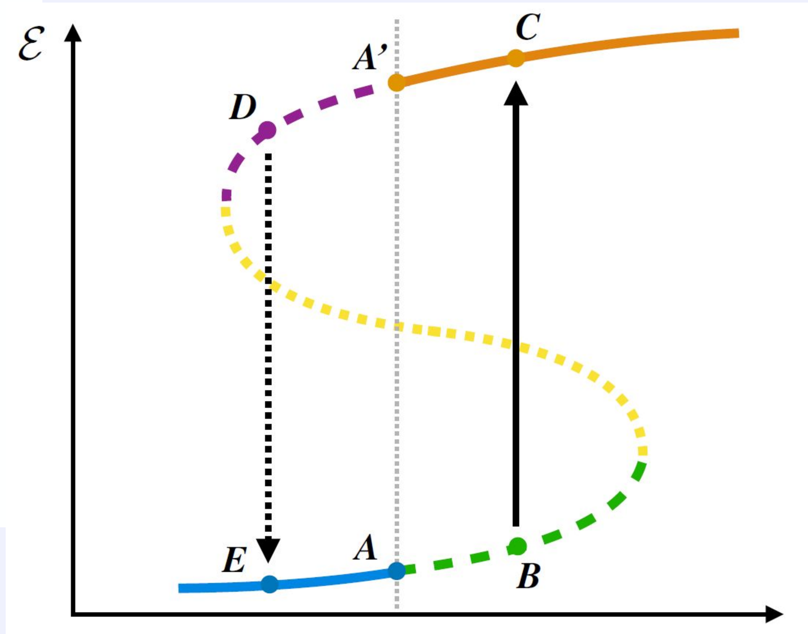
Concise summary

WHAT WE KNOW: our best current description of the particles that make up atoms' nuclei (**protons and neutrons**) is given by Quantum ChromoDynamics, or **QCD**. It says that the nuclear particles are made up of **quarks with gluons** holding them together; and it explains very well most of the phenomena that we can probe via experiments at particle accelerators.

WHAT WE WANT: QCD tells us that at **high temperatures and densities**, protons and neutrons (hadronic phase) should "boil" just like water, undergoing a **phase transition** towards a free gas of quarks and gluons (quark matter phase) with **bubbles** of the metastable phase. However, it does not tell how this happens or at what temperature/density: this is what we want to explore.

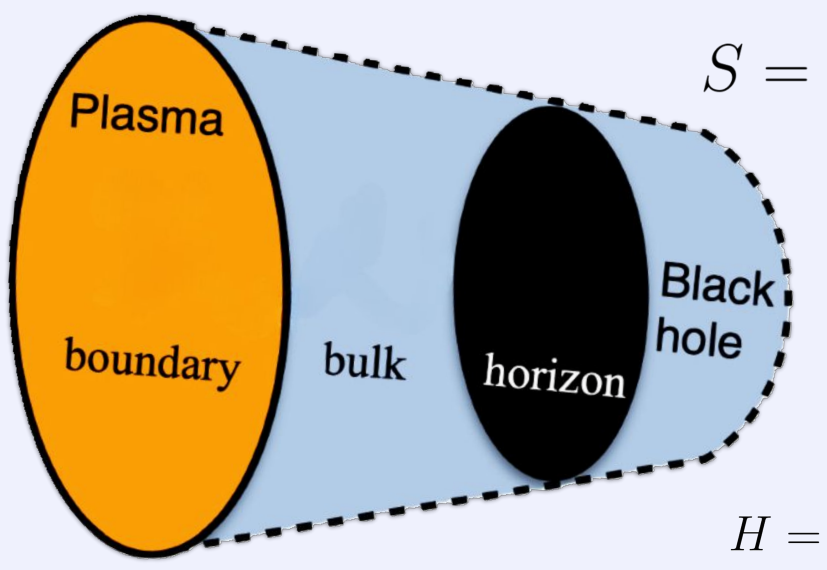
WHY IT'S DIFFICULT: the equations to **simulate the QCD phase transition** at finite density are affected by the so-called "sign problem", which makes it impossible to study via ordinary methods. Thus, we employ String Theory's **Holographic Principle** as a tool, which allows us to make simulations and also compute nonlinear quantities such as the **bubbles' wall velocity**.





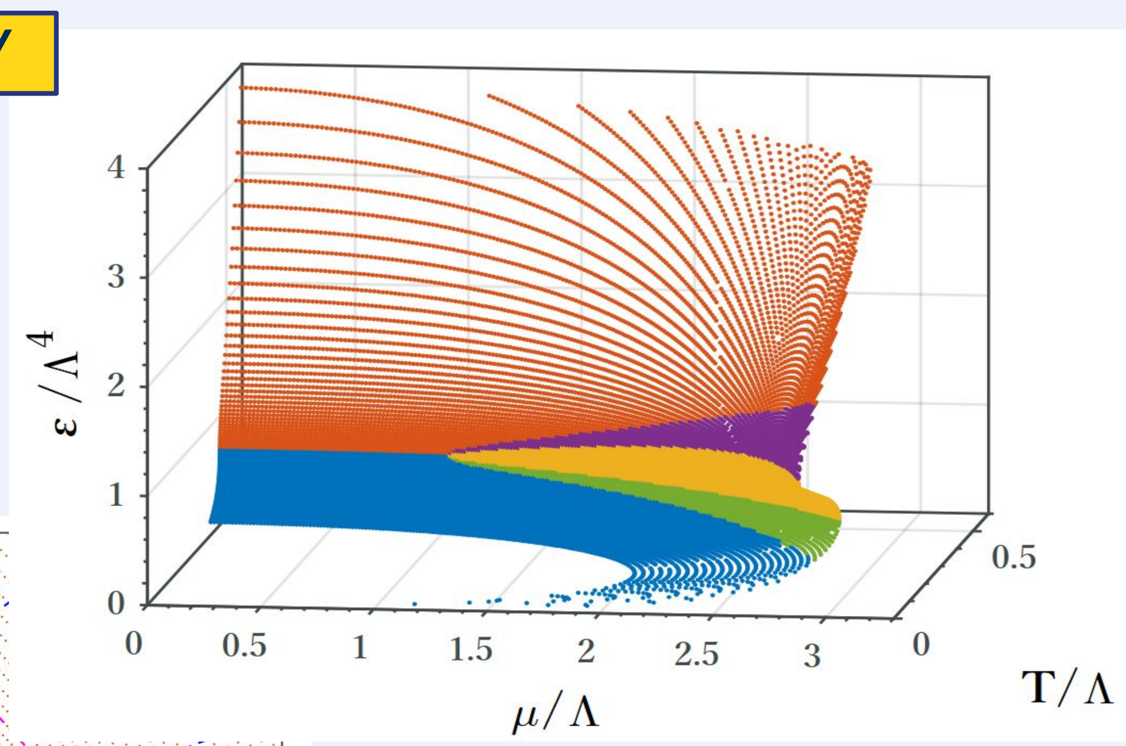
Technical details

1. HOLOGRAPHY and PHASE DIAGRAM STABILITY

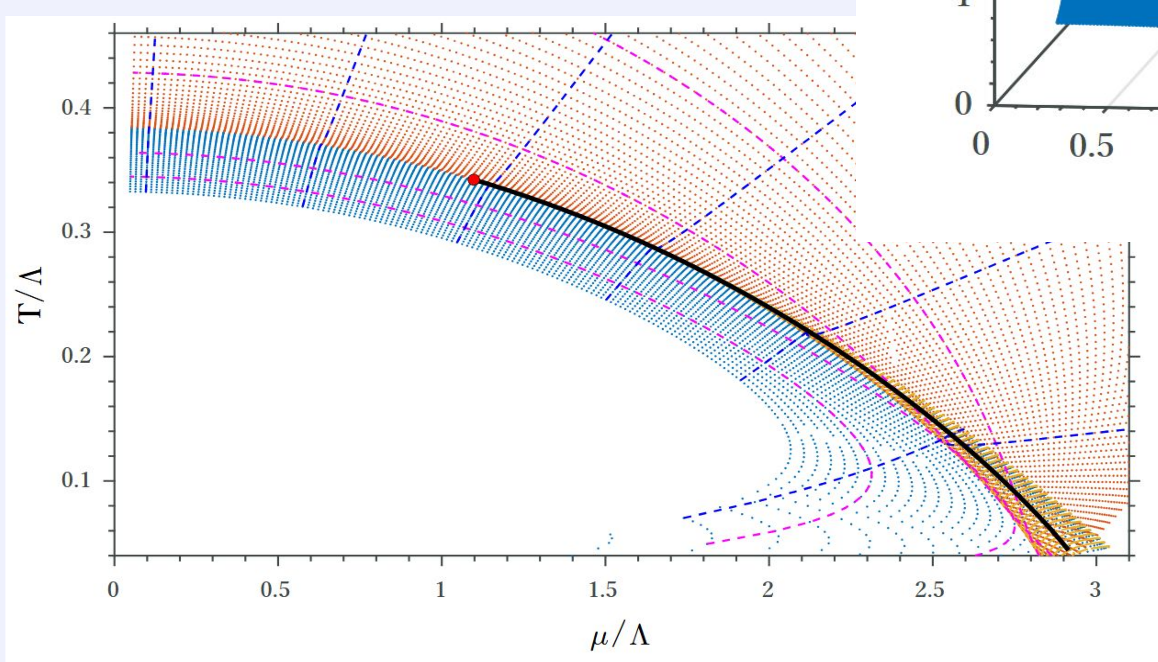


$$S = \frac{2}{\kappa_5^2}\int d^5x\sqrt{-g}\left(\frac{1}{4}\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{f(\phi)}{8}F^2\right)$$

GRAVITY SIDE: Einstein-scalar-Maxwell 5D model
specified by choosing V(ϕ) and f(ϕ). Value of scalar and U(1) fields on the BH horizon completely solve bulk EoM



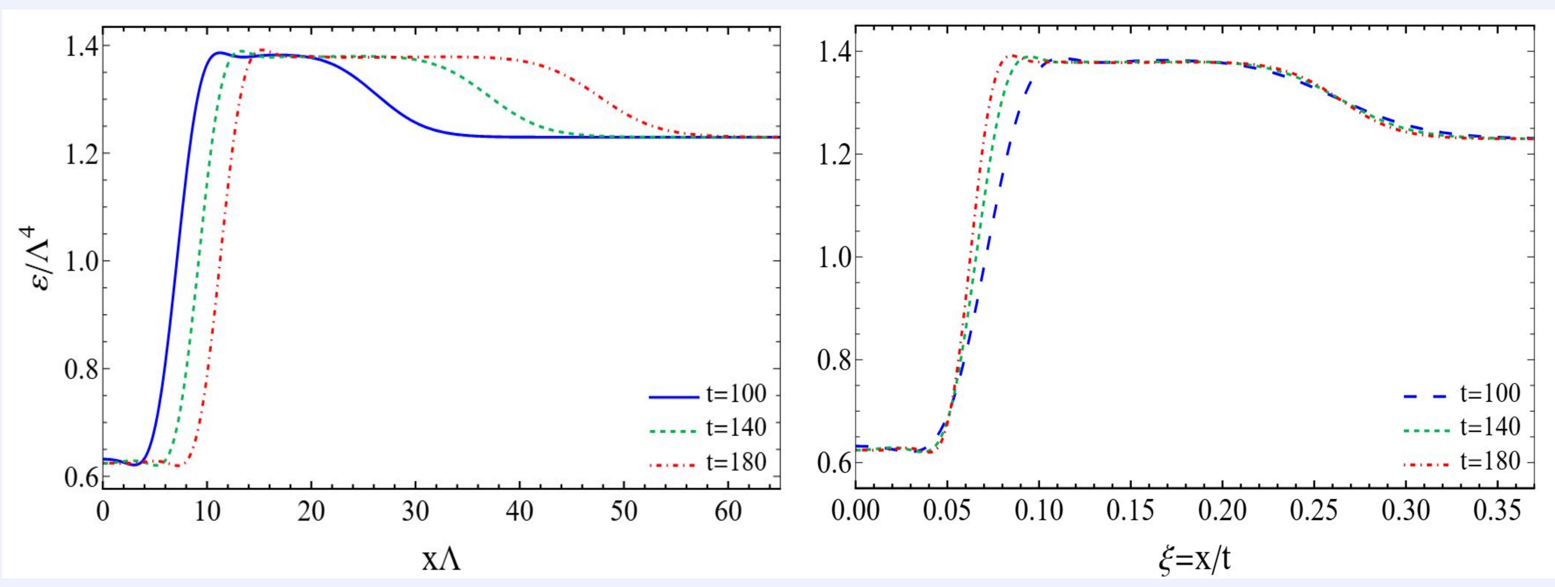
$$H = -\begin{pmatrix} \frac{\partial^2\mathcal{F}}{\partial T^2} & \frac{\partial^2\mathcal{F}}{\partial\mu\partial T} \\ \frac{\partial^2\mathcal{F}}{\partial T\partial\mu} & \frac{\partial^2\mathcal{F}}{\partial\mu^2} \end{pmatrix} = \begin{pmatrix} \frac{\partial s}{\partial T} & \frac{\partial s}{\partial\mu} \\ \frac{\partial\rho}{\partial T} & \frac{\partial\rho}{\partial\mu} \end{pmatrix}$$

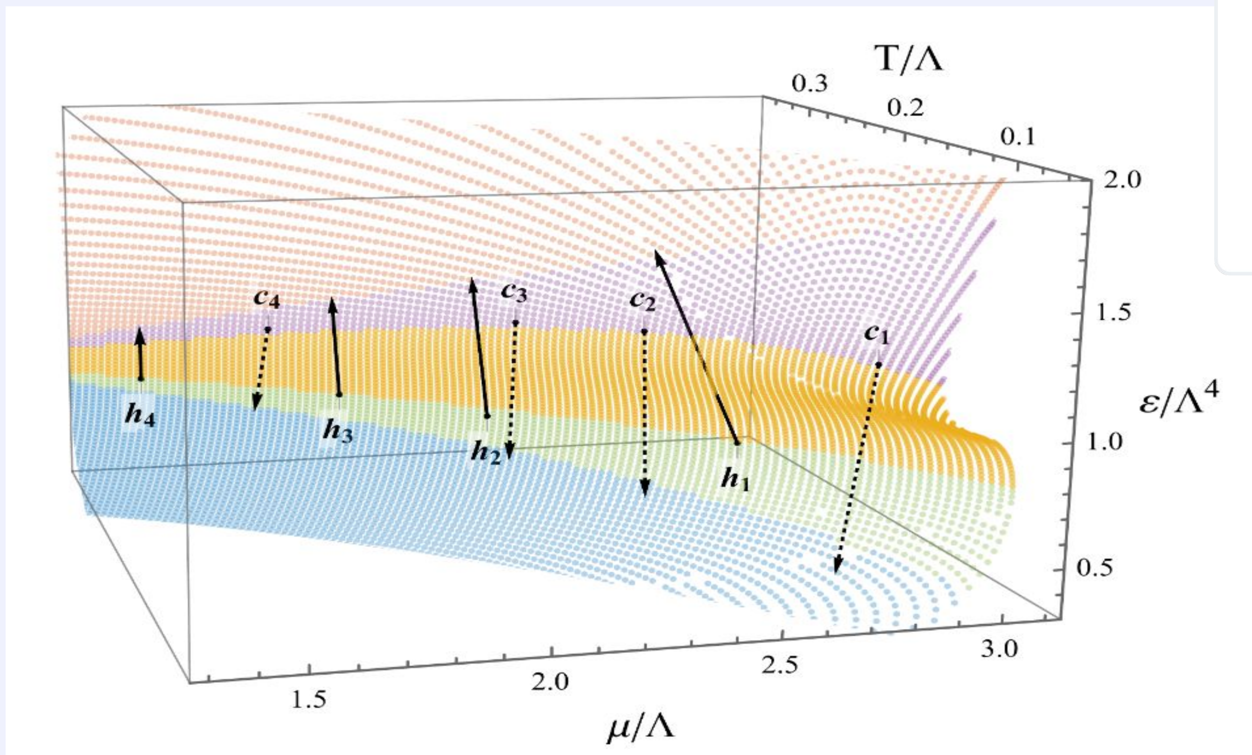


GAUGE SIDE: thermodynamics of QFT
boundary quantities such as entropy, free energy, temperature and pressure are mapped to bulk quantities. With their derivatives, we study (meta-)stability

2. BUBBLE DYNAMICS SIMULATION

Knowing metastable configurations, we use them to simulate a metastable universe which we perturb: the state will evolve towards a stable branch via bubble nucleation.
We study the time evolution in the bulk of expanding, planar bubbles along a chosen boundary direction. We use in-going Eddington-Finkelstein coordinates, with linearized equations solved with `Jecco.jl` (publicly available on GitHub). We read off boundary stress-tensor at different times and positions to analyze the phase transition.





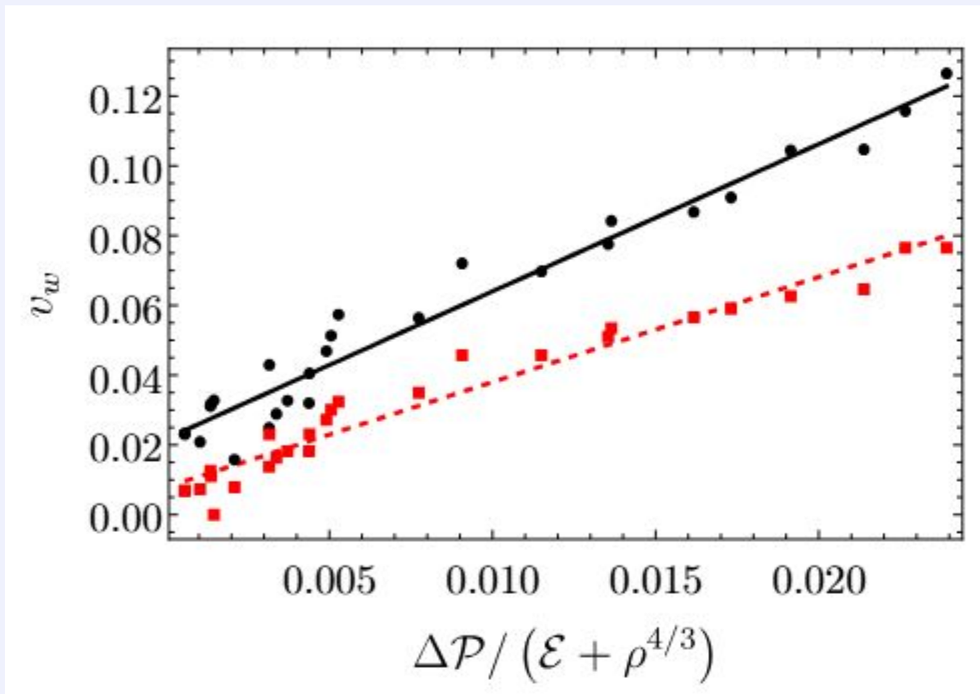
Scan the QR code for a video of the simulated evolution!
In the video, you can see an overcooled bubble: the thermal bath is in the "purple" metastable region and the transition follows the dashed arrow towards the "blue" stable region.

3. WALL VELOCITY ANALYSIS

$$v_w \propto \frac{\Delta\mathcal{P}}{\mathcal{E} + \rho^{4/3}}$$

The bubble wall velocity is a *nonequilibrium parameter*, not obtainable with simple hydrodynamics. We compute it from the simulations when they reach a self-similar steady state, and provide a phenomenological, approximately linear relationship as function of equilibrium parameters (black dots).

We also analyse the predicted expressions for the wall velocity under the assumption that the jump in the number of degrees of freedom across the transition is large (red squares). Due to the specifics of our setup, this prediction works well enough only for slow bubbles.



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