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Investigation Of Rotating Magnetic Brane Solutions Of Quasitopological Gravity In The Presence Of Exponential Nonlinear Field

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Abstract

In this paper, we obtain Rotating magnetic brane solutions of quasi-topological gravity in the presence of exponential nonlinear electrodynamic with one or more rotation parameters. For the rotating brane, the brane has a net electric charge, when one or more rotation are non zero. These solutions are horizonless and have no curvature, but there is a conic singularity with a deficit angle δ. Finally, we analyze the behaviors of the solutions function for the various parameters.

Introduction

Since, many systems in nature that include equations of gravitational systems are inherently nonlinear, in recent years, there has been a great deal of motivation to conduct studies related to quasi-topological gravity and nonlinear electrodynamics. We can name exponential [1] as the type of nonlinear electrodynamics theory, which are defined as

$$L_{exp}(F) = 4\beta^2\left(exp\left(\frac{-F^2}{4\beta^2}\right) - 1\right) \quad (1)$$

$$F^2 = F_{\mu\nu}F^{\mu\nu} \quad (2)$$

β is the nonlinear parameter with dimension of mass and $F = F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu}$ is the electromagnetic field tensor that is defined as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and A_μ is the vector potential. On the other hand, this Lagrangian reduce to the linear Maxwell Lagrangian as $\beta \to \infty$. Like the Born - Infeld theory, exponential nonlinear electrodynamics theory eliminates the infinity of the electric field [2, 3], while the exponential form is unable to do, but this theory causes a weaker singularity than the one in Einstein-Maxwell theory [4]. The exponential form cannot cancel this infinity, but it causes a weaker singularity than the one in Einstein-Maxwell theory [5]. Also, magnetic branes are attractive because their solutions are horizonless and have a conical geometry. Now, we have a decision to take a further step and study the solutions magnetic branes with exponential nonlinear electrodynamic in quartic quasitopological gravity. In this paper, we begin with the metric of a horizonless spacetime and an action including nonlinear electrodynamic and quartic quasitopological theories. Then, we obtain equations and solutions and also, we investigate physical structure and behavior of the obtained solutions. At last, we write a brief result of the obtained data from this magnetic brane.

Rotating Metric And Solutions

We want to abtain the solutions with no horizons. Therefore, we startwith a metric with characteristics $\left(g_{\rho\rho}\right)^{-1} \propto g_{\phi\phi}$ and $g_{tt} \propto -\rho^2$ instead of $\left(g_{\rho\rho}\right)^{-1} \propto g_{tt}$ and $g_{\phi\phi} \propto -\rho^2$. So, we will work with the following rotating metric

$$ds^2 = \left[-\frac{\rho^2}{l^2}\Xi^2 + g(\rho)\Xi^2 - g(\rho)\right]dt^2 + 2\left[\frac{\rho^2}{l}\Xi\sqrt{\Xi^2-1} - lg(\rho)\Xi\sqrt{\Xi^2-1}\right]dtd\phi + \frac{1}{f(\rho)}d\rho^2 + [-\rho^2\Xi^2 + \rho^2 + l^2g(\rho)\Xi^2]d\phi^2 + \frac{\rho^2}{l^2}dX^2 \quad (3)$$

where l is a scale factor that is related to the cosmological constant Λ. $dX^2 = \sum_{i=1}^{n-2}$ is a (n − 2)-dimensional hypersurface with the form of a Euclidean metric in volume V_{n-2}. ρ and of quartic quasi-topological in $(n+1)$-dimensions in the presence of nonlinear ϕ are, respectively, the radial and angular coordinates at which ϕ is dimensionless and has the range$0 \le \phi \le 2\pi$. The action electrodynamic theory is

$$I_G = \int d^{n+1}\sqrt{-g}\left[-2\Lambda + \mathcal{L}_1 + \hat{\lambda}\mathcal{L}_2 + \hat{\mu}\mathcal{L}_3 + \hat{c}\mathcal{L}_4 + L(F)\right] \quad (4)$$

Where g is the determinant of the metric (3) and $\Lambda = -\frac{n(n-1)}{2l^2}$. Just the Einstein-Hilbert lagrangian, second order Lovelock or Gauss-Bonnet lagrangian, cubic and quartic quasi topological Lagrangians are respectively defined as

$$\mathcal{L}_1 = \mathrm{R} \quad (5)$$

$$\mathcal{L}_2 = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2 \quad (6)$$

$$\mathcal{L}_3 = R_{ab}^{cd}R_{cd}^{ef}R_{ef}^{ab} + \frac{1}{(2n-1)(n-3)}\frac{3(3n-5)}{8}R_{abcd}R^{abcd}R - 3(n-1)R_{abcd}R_e^{abc}R^{de} + 3(n+1)R_{abcd}R^{ac}R^{bd} + 6(n-1)R_a^bR_b^cR_c^a - \frac{3(3n-1)}{2}R_a^bR_b^aR + \frac{3(n+1)}{8}R^3 \quad (7)$$

$$\mathcal{L}_4 = c_1R_{abcd}R^{cdef}R_{ef}^{hg}R_{hg}^{ab} + c_2R_{abcd}R^{abcd}R_{ef}R^{ef} + c_3RR_{ab}R^{ac}R_c^b + c_4\left(R_{abcd}R^{abcd}\right)^2 + c_5R_{ab}R^{ac}R_{cd}R^{db} + c_6RR_{abcd}R^{ac}R^{db} + c_7R_{abcd}R^{ac}R^{be}R_e^d + c_8R_{abcd}R^{acef}R_e^bR_f^d + c_9R_{abcd}R^{ac}R_{ef}R^{bedf} + c_{10}R^4 + c_{11}R_{abcd}R^{abcd}R^2 + c_{12}R_{ab}R^{ab}R^2 + c_{13}R_{abcd}R^{cbef} + c_{14}R_{abcd}R^{acef}R_{gehf} \quad (8)$$

$$c_1 = -(n-1)(n^7 - 3n^6 - 29n^5 + 170n^4 - 349n^3 + 348n^2 - 180n + 36)$$
$$c_2 = -4(n-3)(2n^6 - 20n^5 + 65n^4 - 81n^3 + 13n^2 + 45n - 18)$$
$$c_3 = -64(n-1)(3n^2 - 8n + 3)(n^2 - 3n + 3)$$
$$c_4 = -(n^8 - 6n^7 + 12n^6 - 22n^5 + 114n^4 - 345n^3 + 468n^2 - 270n + 54)$$
$$c_5 = 16(n-1)(10n^4 - 51n^3 + 93n^2 - 72n + 18)$$
$$c_6 = 32(n-1)^2(n-3)^2(3n^2 - 8n + 3)$$
$$c_7 = 64(n-2)(n-1)^2(4n^3 - 18n^2 + 27n - 9)$$
$$c_8 = -96(n-1)(n-2)(2n^4 - 7n^3 + 4n^2 + 6n - 3)$$
$$c_9 = 16(n-1)^3(2n^4 - 26n^3 + 93n^2 - 117n + 36)$$
$$c_{10} = n^5 - 31n^4 + 168n^3 - 360n^2 + 330n - 90$$
$$c_{11} = 2(6n^6 - 67n^5 + 311n^4 - 742n^3 + 936n^2 - 576n + 126)$$
$$c_{12} = 8(7n^5 - 47n^4 + 121n^3 - 141n^2 + 63n - 9)$$
$$c_{13} = 16n(n-1)(n-2)(n-3)(3n^2 - 8n + 3)$$
$$c_{14} = 8(n-1)(n^7 - 4n^6 - 15n^5 + 122n^4 - 287n^3 + 297n^2 - 126n + 18) \quad (9)$$

$\hat{\lambda}$, $\hat{\mu}$ and $\hat{c}$ are respectively the parameters of Gauss-Bonnet, cubic and quartic quasi- topological Lagrangians. Actually, these parameters are arbitrary coupling constants, and rescaling is done to simplify the field equations. So, gives [6, 7]

$$\hat{\lambda} = \frac{\lambda l^2}{(n-2)(n-3)} \quad (10)$$

$$\hat{\mu} = \frac{7(2n-1)l^4\mu}{(n-2)(n-5)(3n^2-9n+4)} \quad (11)$$

$$\hat{c} = \frac{cl^6}{n(n-1)(n-3)(n-7)(n-2)^2(n^5-15n^4+72n^3-156n^2+150n-42)} \quad (12)$$

For the static magnetic brane, the vector potential has only one component A_ϕ, while for the rotating magnetic brane, add angular momentum to the spacetime. So, in this class, the vector potential includes two components A_ϕ and A_t. Therefore, the vector potential for the rotating solutions as

$$A_\mu = h(\rho)\left(\frac{\sqrt{\Xi^2-1}}{l}\delta_\mu^t - \Xi\delta_\mu^\phi\right) \quad (13)$$

Using the above relations in action (4) and integrating by parts, we can get the action

$$S = \frac{(n-1)}{16\pi l^2}\int d^nx\int d\rho N(\rho)\left[[\rho^n(1 + \psi + \lambda\psi^2 + \mu\psi^3 + c\psi^4)]' + \frac{4\beta^2l^2\rho^{n-1}}{n-1}\left[exp\left(-\frac{h'^2}{2\beta^2l^2N^2(\rho)}\right) - 1\right]\right] \quad (14)$$

where $g(\rho) = N(\rho)^2f(\rho)$, $\psi(\rho) = -l^2\rho^{-2}f(\rho)$ and prime represents the first derivative with respect to ρ. Varying this action with respect to $\psi(\rho)$ yields

$$[1 + 2\lambda\psi(\rho) + 3\mu\psi^2(\rho) + 4c\psi^3(\rho)]N'(\rho) = 0 \quad (15)$$

The above equation shows that $N(\rho)$ must be a constant value, which we choose $N(\rho) = 1$. Varying the action 14 with respect to $N(\rho)$ and $h(\rho)$ and substituting $N(\rho)$=1 (or $g(\rho) = f(\rho)$) yield

$$[(n-1)\rho^n((1 + \psi + \lambda\psi^2 + \mu\psi^3 + c\psi^4)]' + \rho^{n-1}\left(l^2\beta^2 + h'^2\right)exp\left(-\frac{h'^2}{2l^2\beta^2}\right) - 4l^2\beta^2\rho^{n-1} = 0 \quad (16)$$

$$\left(\rho^{n-1}h'\exp\left(-\frac{h'^2}{2l^2\beta^2}\right)\right)' = 0 \quad (17)$$

If we solve Eq. (17), we get the electromagnetic field

$$F_{\phi\rho} = \Xi h' = \Xi l\beta\sqrt{-Lw(-\eta)} \quad (18)$$

and

$$F_{t\rho} = \frac{\sqrt{\Xi^2-1}}{l\Xi}F_{\phi\rho} = \Xi h' = -l\beta\frac{\sqrt{\Xi^2-1}}{l}\sqrt{-Lw(-\eta)} \quad (19)$$

where $\eta = \frac{q^2l^{2n-6}}{\beta^2\rho^{2n-2}}$ and q is the constant of integration. Lw is the Lambert function

We get to the relation $F_{\phi\rho} = -\partial_\rho A_\phi$ and $F_{t\rho} = -\partial_\rho A_t$.

$$A_\phi = -\Xi\frac{n-1}{n-2}l\beta\left(\frac{l^{n-3}q}{\beta}\right)^{\frac{1}{n-1}}(-Lw(-\eta))^{\frac{n-2}{2(n-1)}} \times \left\{2^{F_1}\left(\left[\frac{n-2}{2(n-1)}\right], \left[\frac{3n-4}{2(n-1)}\right], -\frac{1}{2(n-1)}Lw(-\eta)\right)\right. \quad (20)$$

Using equation 18 in equation 16 leads to the relation

$$c\psi^4 + \mu\psi^3 + \lambda\psi^2 + \psi + k = 0 \quad (21)$$

where k is

$$k = 1 - \frac{M}{(n-1)\rho^n} + -\frac{4l^2\beta^2}{n(n-1)} - \frac{4(n-1)q\beta l^{n-1}}{n(n-2)\rho^n}\left(\frac{l^{n-3}q}{\beta}\right)^{\frac{1}{n-1}}(-Lw(-\eta))^{\frac{n-2}{2(n-1)}} \times 2^{F_1}\left(\left[\frac{n-2}{2(n-1)}\right], \left[\frac{3n-4}{2(n-1)}\right], -\frac{1}{2(n-1)}Lw(-\eta)\right) + \frac{4\beta ql^{n-1}}{(n-1)\rho^{n-1}}(-Lw(-\eta))^{\frac{1}{2}} \times \left[1 + \frac{1}{n}(-Lw(-\eta))^{-1}\right] \quad (22)$$

and M is the integration constant and is related to the mass of this magnetic brane. In the above solution, we have used the following relation for the Lambert function:

$$Lw(x)e^{Lw(x)} = x \quad (23)$$

Finally, the solution $f(\rho)$ for Eq. (21) is obtained as

$$f(\rho) = -\frac{\rho^2}{l^2}\left(-\frac{\mu}{4c} + \frac{\pm_sW\mp_t\sqrt{-(3\alpha+2y\pm_s\frac{2\beta}{W})}}{2}\right) \quad (24)$$

In the above equation, two $\pm_s$ should both have the same sign, while the sign of $\mp_t$ is independent. It is noteworthy to say that in order to have the cubic quasi-topological or Gauss-Bonnet solutions we should replace $\mu = 0$ or $\lambda = 0$ in Eq. (21) and not find the solutions in the above relations because we get vague values [6,8,9].

Physical Properties Of The Solutions

In this section, we aim to investigate the geometric and physical properties of the solutions like horizons, singularity, and behaviors of the function $f(\rho)$. As we know, to find the horizons of the obtained solutions, the condition $f(r_+) = 0$ should be satisfied where r_+ is the horizon. Suppose that r_+ is the largest real root of $f(\rho) = 0$, which leads to the function $f(\rho)$ being positive for $\rho > r_+$and negative for $\rho < r_+$. The range $0 < \rho < r_+$ is not acceptable as $g_{\rho\rho}$ cannot be negative (which occurs for $\rho < r_+$, because of the change of signature of the metric from $(n-1)+$ to $(n-2)+$. Therefore, we delete this unacceptable range $0 < \rho < r_+$, and so the function $f(\rho)$ is limited to the acceptable range $\rho > r_+$. For ease, we can use the suitable transformation

$$\mathrm{r}^2 = \rho^2 - \mathrm{r}_+^2 \quad (25)$$

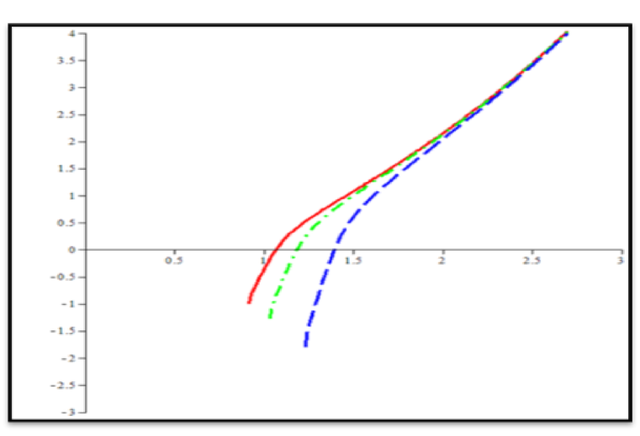

Figure 1. $f(\rho)$ versus ρ with $M = 5$, $\beta = 10$, $n = 4$, $\lambda = -0/01$, $\mu = 0.4$, and $c = -0.01$

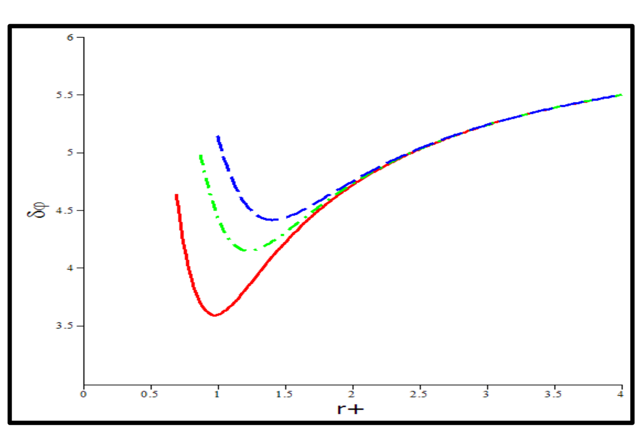

Figure 2.$\delta\phi$versus$r_+$with $\beta = 10$ and $n = 4$.

In Fig. 1, $f(\rho)$ versus ρ for different values of q and for Exponential nonlinear electrodynamic is plotted. As mentioned, there is a r_+ which $f(\rho) < 0$ for $\rho < r_+$ and unacceptable. Also, for constant values of parameters M, β, n, λ, μ and c, when we increase the value of q, the value of r_+ increases.The function f has a constant value for each value of ρ, but in the region near r_+, as $q$increases it decreases. In Fig. 2, for different values of q, there is a minimum value for r_+ that $\delta\phi$ is real only for $r_+ > r_{+min}$ and the same way for $r_+ > r_{+max}$, $\delta\phi$ is independent of q and has a constant value for each value of r_+. While in the range of $r_{+min} < r_+ < r_{+max}$, $\delta\phi$ depends on the value of q and as q increases it increases.

Concluding Results

In this paper, we obtained magnetic solutions of quartic quasi-topological gravity in the presence of nonlinear electrodynamic exponential generated by a rotating magnetic brane. It should be noted that quasi-topological gravity is a higher derivative theory and has no limitations on dimensions. So, if we consider the parameters of quasi-topological gravity zero ($\lambda = \mu = c = 0$) this theory reduces to Einstein's theory and also reduces to linear Maxwell field, if the nonlinearity parameter β goes to infinity. This solutions have no curvature singularity and no horizons, but have conical singularity with a deficit angle. The function f is defined in the range $r_+ < \rho < \infty$ and does not contain the point $\rho = 0$. In the continue, we analyze the behaviors of the function f for the various parameters.

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